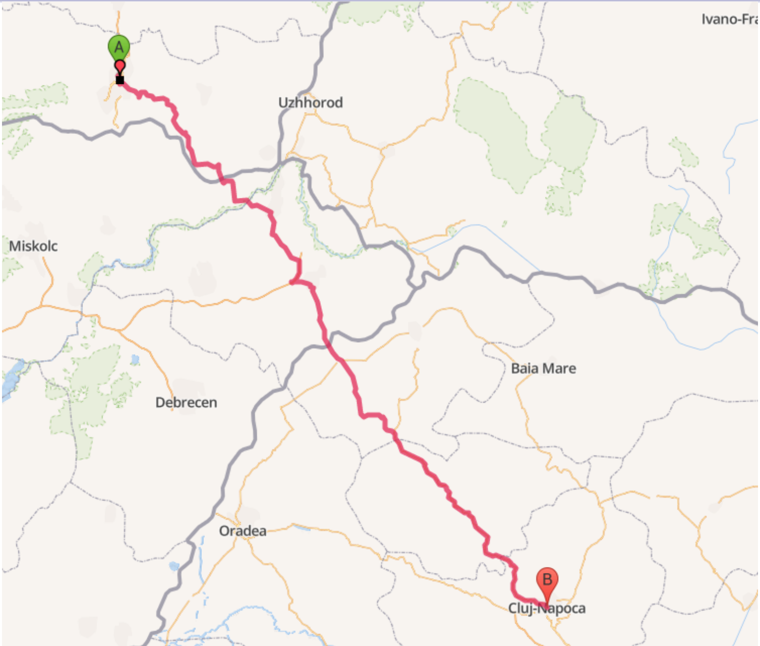
Description Logics

Adrian Groza

Department of Computer Science
Technical University of Cluj-Napoca



A
B
Uzhhorod
Ivano-Fra
Miskolc
Baia Mare
Debrecen
Oradea
Cluj-Napoca

Outline

Semantic Web Vision

"The Semantic Web is not a separate Web but an extension of the current one, in which information is given well-defined meaning, better enabling computers and people to work in cooperation."

Tim Barners Lee

Social Web

Semantic Web

Semantic Search

What would you like to know?

Which is the population of Cluj-Napoca?

Which is the population of Cluj-N

306,474

website wikipedia

What would you like to know?

Is Alice Walton richer than Christy Walton?

Is Alice Walton richer than Christy

No

Alice Walton
Alice Walton (born 1949)
wikipedia

Christy Walton
Christy Ruth Walton, the wife of late John T. Walton
wikipedia

What would you like to know?

Is Paris Hilton single?

Is Paris Hilto

Yes

Paris Hilton
Paris Whitney Hilton (born February 17, 1981), the American socialite, media personality, model, singer and actress
website wikipedia

▼ How do we know?

✓ *facts...* See reason

I used the following facts to provide this answer:

'is richer than' is treat as antisymmetric	agree	disagree
human being, organisation or other legal person is the left class of 'is richer than'	agree	disagree
human being, organisation or other legal person is the right class of 'is richer than'	agree	disagree
'is richer than' is a left comparison of 'is the net worth of'	agree	disagree
$19,200,000,000 has been the net worth of Christy Walton since at least February 11th 2008	agree	disagree
$19,000,000,000 has been the net worth of Alice Walton since at least February 11th 2008	agree	disagree
United States dollar is an unit of currency	agree	disagree

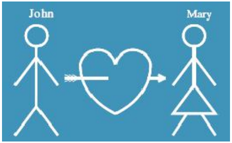
A Logician's View of the World

- Let's start for the very beginning:

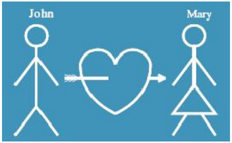
John loves Mary

- But let's view it in a more graphical way:

Something important is missing



And now we said too much!!!



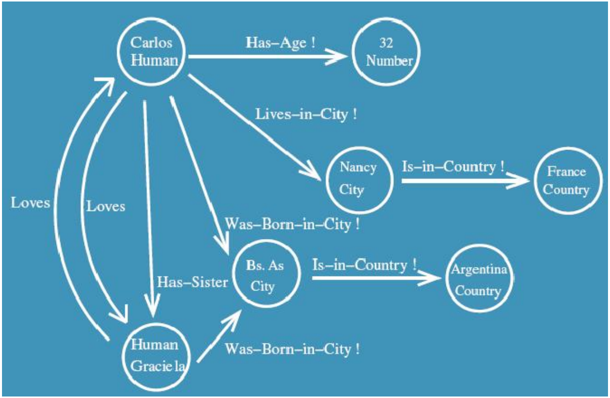
- Who said that John was a man?
- Who said that Mary was wearing a skirt?
- And what does a heart to do with love?

What we know is that:

- There is something called John
- There is something called Mary
- There is something called Love going on between John and Mary

How far can we go?

Carlos is 32 years old



Now for Something More Complicated

- Suppose we want to say ”something general”, like

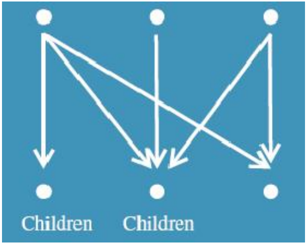
  Grandparents Love Children

- Notice that we want to establish a relation between two sets: the set of Grandparents and the set of Children, not just about two individuals like *John* and *Mary*
- So we have to think in terms of sets

  Grandparents is a subset of the set of things that love Children

Some Ambiguity and Some Notation

- How do we construct, given the set of Children, the set of all those who love them?
- But what do we mean by ”Those who love children”? If I’m a member of this set, should I:



1. Love all children
2. Love at least one child
3. Love only children

- Standard notation
  $\exists love.Children$
  $\forall love.Children$

Terminologies

General Inclusion Axioms

Coming back to the Grandparents we can now write:
$Grandparents \sqsubseteq \exists love.Children$

Definitions

If we can represent subsets, then we can also express equivalence (using double inclusion):
$GP \equiv Human \sqcap \exists hasChild.\exists hasChild.Human \sqcap \exists love.Children$

Outline

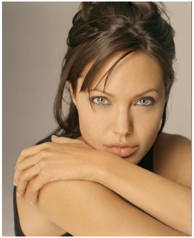
Let's Try Our Hand with Some Definitions!

Person
Female
Woman $=$
Man $=$
Mother $=$
Father $=$
Parent $=$
Grandmather $=$
Wife $=$
MotherWithoutDaugther

$Person$
$Female$
$Woman = Person \sqcap Female$
$Man = Person \sqcap \neg Female$
$Mother = Woman \sqcap \exists hasChild.Person$
$Father = Man \sqcap \exists hasChild.Person$
$Parent = Father \sqcup Mother$
$Grandmather = Mother \sqcap \exists hasChild.Parent$
$Wife = Woman \sqcap \exists hasHusband.Man$
$MotherWithoutDaugther =$
$Mother \sqcap \exists hasChild.\neg Woman$

More for ”Family” Ontology

We will define the concept of ”lucky man” as a man who has a rich and beautiful wife and all his children are happy.

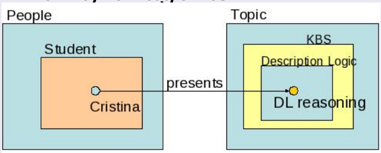


$$LuckyMan = Man \sqcap \exists married.(Rich \sqcap Beautiful) \sqcap \forall hasChild.Happy$$

An ontology of this talk

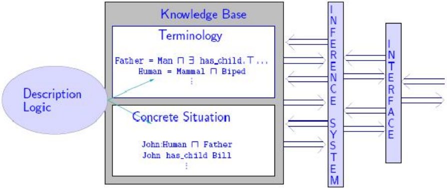
Ontologies

Description of the concepts and relationships that can exist for an agent or a community of agents.



$$Participant \sqcap \exists attends.Talk$$
$$Participant \sqcap \forall attends.(Talk \sqcap \neg Boring)$$
$$Speaker \sqcap \exists gives.(Talk \sqcap \forall topic.DL)$$
$$Speaker \sqcap \exists gives.(Talk \sqcap \forall topic.(DL \sqcup Ontologies))$$

Architecture of a Standard DL System



Terminological part (Tbox)

- Describes the notions by stating properties on concepts and roles and relationships between them
- Abreviations: $LuckyMan \equiv Man \sqcap \exists married.(Rich \sqcap Beautiful)$
- General axioms: $\exists hasChild.Human \sqsubseteq Human$

Assertional part (Abox)

- Describes concrete situation: $brad : Father$, $carlos : \neg Father bob : HappyMan$, $(bob, mary) : hasChild$, $\neg mary : \neg Doctor$

DL Representatives - Attributive Language (AL)

$C, D \rightarrow$	A	(atomic concept)
	$\top$	(universal concept)
	$\bot$	(bottom concept)
	$\neg A$	(atomic negation)
	$\sqcap$	(intersection)
	$\forall R.C$	(value restriction)
	$\exists R.C$	(existential quantification)

Example (Syntax of AL)

- $Person \sqcap Female$, $Person \sqcap \neg Female$
- Those persons that have a child: $Person \sqcap \exists hasChild.\top$
- Those persons all of whose children are female: $Person \sqcap \forall hasChild.Female$
- Those persons without a child: $Person \sqcap \forall hasChild.\bot$

AL with Complements: $C \sqcup D = \neg(\neg C \sqcap \neg D)$

Number restrictions

Example (Number restrictions)

A person can be married to at most one other individual:

$$Person \sqsubseteq \leq 1 married.\top$$

Instances of *HappyMan* have between two and four children:

$$HappyMan \equiv Human \sqcap \geq 2 hasChild.\top \sqcap \leq 4 hasChild.\top$$

Inverse Roles

Example (Inverse roles)

- $hasChild^{-} = hasParent$

$$hasParent(Bob, Alice) \rightarrow hasChild(Alice, Bob)$$

- Presenter of a boring talk:
  $Speaker \sqcap gives.(Talk \sqcap \forall attends^{-1}.(Bored \sqcup Sleeping))$

Semantics of r^{-}

$(r^{-})^{\mathcal{I}} = \{(b, a) | (a, b) \in r^{\mathcal{I}}$

Nominals

Example (All computer scientistis that have met Turing)

$$ComputerScientist \sqcap \exists hasMet.\{Turing\}$$

A nominal is a singleton set

$\{a\}^{\mathcal{I}} = \{a^{\mathcal{I}}\}$

Example (Express ABox assertions through GCIs)

$C(a) \Leftrightarrow \{a\} \sqsubseteq C$
$r(a, b) \Leftrightarrow \{a\} \sqsubseteq \exists r.\{b\}$

One of

extends nominal constructor to a finite set of individuals.

Role constraints

- (Transitive Role) $hasBrother^+$

  $hasBrother(Bob, David), hasBrother(David, Mack) \rightarrow hasBrother(Bob, Mack)$

- $(RoleHierarchy) hasMother \sqsubseteq hasParent$

  $$hasMother(Bob, Alice) \rightarrow hasParent(Bob, Alice)$$

  $married \sqsubseteq loves$

Domain and range restrictions

Example (Domain restriction)

Only human beings can have human children:
$\exists child.Human \sqsubseteq Human$
Only parents can have children: $\exists hasChild.\top \sqsubseteq Parent$

Domain restriction $dom(r) \sqsubseteq C$

$\exists r.\top \sqsubseteq C$

Example (Range restriction)

The child of a human being must be human:
$Human \sqsubseteq \forall child.Human$

Range restriction $ran(r) \sqsubseteq C$

$\top \sqsubseteq \forall r.C$

Extensions

Disjointness of concepts

$Woman \sqcap Man \equiv \bot$.

Example (Reflexivity)

Every entity is part of itself: $partOf^r$

Reflexive roles

Every individual is related to itself

Roles

Transitive	hasAncestor	$R(a,b)$ and $R(b,c) \rightarrow R(a,c)$
Symmetric	hasSpouse	$R(a,b) \rightarrow R(b,a)$
Asymmetric	hasChild	$R(a,b) \rightarrow$ not $R(b,a)$
Reflexive	hasRelative	$R(a,a)$ for all a
Irreflexive	parentOf	not $R(a,a)$ for any a
Functional	hasHusband	$R(a,b)$ and $R(a,c) \rightarrow b = c$
InverseFunctional	hasHusband	$R(a,b)$ and $R(c,b) \rightarrow a = c$

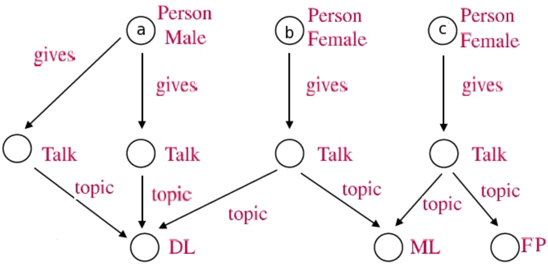
Extensions of ALC

SHOIQ = OWL DL

- S=ALCR+: ALC with transitive role
- H = role hierarchy (subrole)
- O = nominal .e.g WeekEnd = $\{$Saturday, Sunday$\}$
- I = Inverse role
- Q = qulified number restriction e.g. $\geq 1 hasChild.Man$
- N = number restriction e.g. $\geq 1 hasChild$
- D = concrete domains $\Rightarrow$ SHOIQ (D)

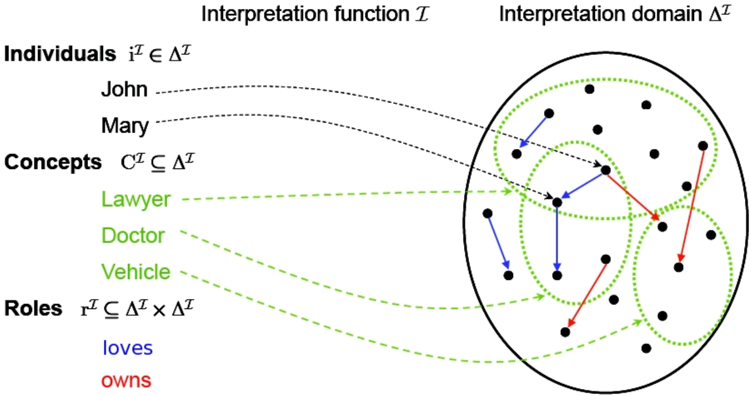
Outline

Example of an interpretation



- $(Person \sqcap \exists gives.(Talk \sqcap \forall topic.DL))^{\mathcal{I}}$
- $(Person \sqcap \exists gives.(Talk \sqcap \exists topic.DL))^{\mathcal{I}}$
- $(Female \sqcap \exists gives.(Talk \sqcap \exists topic.DL))^{\mathcal{I}}$

Semantics



The interpretation $\mathcal{I}$ is a model of the tbox $\mathcal{T}$ iff it satifies all the GCI in $\mathcal{T}$.
Two tboxes are called equivalent if they have the same models.

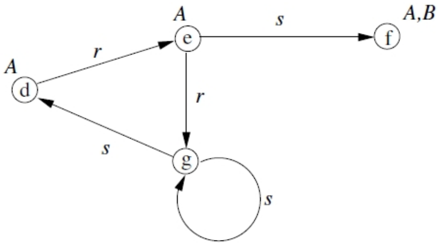
Semantics

$\Delta = \{t_1, t_2, f_1, f_2, c_1, c_2, j, k, l, m, n\}$
$Person^{\mathcal{I}} = \{j, k, l, m, n\}$
$Car^{\mathcal{I}} = \{t_1, t_2, f_1, f_2, c_1, c_2\}$
$Ferrari^{\mathcal{I}} = \{f_1, f_2\}$
$Toyota^{\mathcal{I}} = \{t_1, t_2\}$
$likes^{\mathcal{I}} =$
$\{(j, f_1), (k, f_1), (k, t_2), (l, c_1), (l, c_2), (m, c_1), (m, t_2), (n, f_2), (n, c_2)\}$
Find the interpretation in $\mathcal{I}$ of the following concepts:

- $\exists likes.Ferrari \sqcap \exists likes.Toyota$
- $\exists likes.Ferrari \sqcap \forall likes.Ferrari$
- $\exists likes.Ferrari \sqcap \exists likes.\neg Ferrari$
- $\exists likes.Cars \sqcap \forall likes.\neg(Toyota \sqcup Ferrari)$

Semantics

Consider the (graphical representation of the) interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{d, e, f, g\}$:



List all elements x of $\Delta^{\mathcal{I}}$ such that $x \in C^{\mathcal{I}}$:

- $A \sqcup B$
- $\exists s.\neg A$
- $\exists s.A \sqcap \forall s.A$
- $\exists s.\exists s.\exists s.\exists s.A$
- $\neg\forall r.(\neg A \sqcap \neg B)$

Outline

Basic Inference problems

Subsumption: $C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in all models $\mathcal{I}$ of O.
Equivalence: $C \equiv D$ iff $C^{\mathcal{I}} = D^{\mathcal{I}}$ in all models $\mathcal{I}$ of O.
Satisfiability: $C \not\equiv \bot$ iff $C^{\mathcal{I}}$ non empty in some model $\mathcal{I}$ of O.
Instantiation: $i \in C$ iff $i \in C^{\mathcal{I}}$ in all models $\mathcal{I}$ of O
Consistency: O consistent iff there is at least one model I of O.
Coherency: O coherent iff all concepts are satisfiable.
Problems reduced to satisfiability: $C \sqsubseteq D$ iff $C \sqcap \neg D$ not satisfiable.

Inference services - examples

Example (Subsumption)

$HappyMan = Human \sqcap \neg Female \sqcap (\exists married.Doctor) \sqcap$
$(\forall hasChild.(Doctor \sqcup Professor))$
$Doctor \sqsubseteq Human$
$HappyMan$ is subsummed by $\exists married.Human$

Example (Instance)

$HappyMan(bob), hasChild(bob, mary), \neg Doctor(mary)$
$mary : Professor$

Example (Consistency)

$HappyMan(bob), hasChild(bob, mary),$
$\neg Doctor(mary), \neg Professor(mary)$

NNF: negation only in front of atomic classes.

1. Replace $C \equiv D$ by $C \sqsubseteq D$ and $D \sqsubseteq C$
2. Replace $C \sqsubseteq D$ by $\neg C \sqcup D$
3. Apply the equations:

$$
\begin{aligned}
\mathrm{NNF}(C) &= C \quad \text{if } C \text{ is a class name} \\
\mathrm{NNF}(\neg C) &= \neg C \quad \text{if } C \text{ is a class name} \\
\mathrm{NNF}(\neg\neg C) &= \mathrm{NNF}(C) \\
\mathrm{NNF}(C \sqcup D) &= \mathrm{NNF}(C) \sqcup \mathrm{NNF}(D) \\
\mathrm{NNF}(C \sqcap D) &= \mathrm{NNF}(C) \sqcap \mathrm{NNF}(D) \\
\mathrm{NNF}(\neg(C \sqcup D)) &= \mathrm{NNF}(\neg C) \sqcap \mathrm{NNF}(\neg D) \\
\mathrm{NNF}(\neg(C \sqcap D)) &= \mathrm{NNF}(\neg C) \sqcup \mathrm{NNF}(\neg D) \\
\mathrm{NNF}(\forall R.C) &= \forall R.\mathrm{NNF}(C) \\
\mathrm{NNF}(\exists R.C) &= \exists R.\mathrm{NNF}(C) \\
\mathrm{NNF}(\neg\forall R.C) &= \exists R.\mathrm{NNF}(\neg C) \\
\mathrm{NNF}(\neg\exists R.C) &= \forall R.\mathrm{NNF}(\neg C)
\end{aligned}
$$

Example ($P \sqsubseteq (E \sqcap U) \sqcup \neg(\neg E \sqcup D)$)

$\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D)$

Reduction to (un)satisfiability

Idea: given knowledge base K

- Attempt construction of a tree (called Tableau), which represents a model of K.
- If attempt fails, K is unsatisfiable.

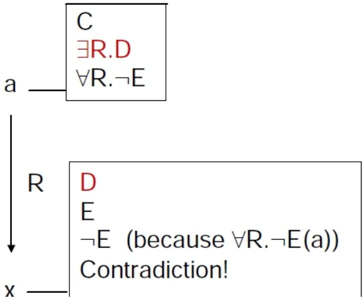
The tableau is a directed labeled graph:

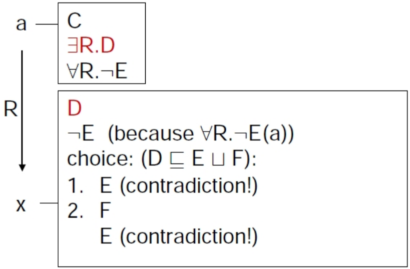
- Nodes represent elements of the domain of the model. Every node x is labeled with a set $L(x)$ of concepts: $C \in L\{x\} \equiv$ "x is in the extension of C".
- Edges stand for role relationships. Every edge $\prec x, y \succ$ is labeled with a set $L(\prec x, y \succ)$ of role names. $R \in L(\prec x, y \succ) \equiv$ "(x, y) is in the extension of R".

Example ($C(a)$, $C \sqsubseteq \exists R.D$, $D \sqsubseteq E$)

Does this entail $(\exists R.E)(a)$?
Add $\forall R.\neg E(a)$ and show unsatisfiability:

Tableaux example





Example ($C(a)$, $C \sqsubseteq \exists R.D$, $D \sqsubseteq E \sqcup F$, $F \sqsubseteq E$)

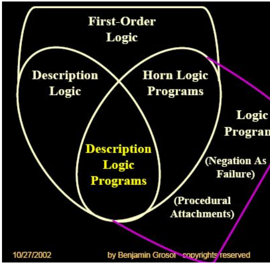
Does this entail $(\exists R.E)(a)$?
Add $\forall R.\neg E(a)$ and show unsatisfiability:

Outline

$DL \setminus LP \sqcup LP \setminus DL$

Examples of DL beyond DLP

1. State a subclass of a complex class expression which is a disjunction $(Human \sqcap Adult) \sqsubseteq (Man \sqcup Woman)$
2. State a subclass of a complex class expression which is an existential $Radio \sqsubseteq \exists hasPart.Tuner$
   Why not? Because: LP/Horn, cannot represent a disjunction or existential in the head



Examples of LP beyond DLP

- A rule involving multiple variables.

$Man(X) \wedge Woman(Y) \rightarrow PotentialLoveInterestBetween(X, Y)$

DL's not used to represent "more than one free variable at a time"

References

- Acknowledgement: Slides adapted from Jie Bao, Carlos Areces
- Reading: Knowledge Representation and Reasoning, Logic meets Probability Theory, Peter Lucas, pages 39-43
- Additional Reading: The Description Logic Handbook Theory, Implementation and Applications Edited by Franz Baader, Diego Calvanese, Deborah McGuinness, Daniele Nardi, Peter Patel-Schneider, Chapter 2, Basic Description Logics